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A Modification of the Method for Defining and Measuring Crimp of Textile Fibers[1]

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Abstract

The main parameter used to describe the geometry of the crimp of a fiber is the *crimp diameter*. This corresponds to an average amplitude of the fiber in space while the fiber is kept in a static position of stress.

In order to measure the crimp diameter, it is necessary to measure the coordinates of a sufficient number of points on the fibers. From these measurements, a "fiber axis" can be defined as the straight line for which the sum of squares, S, of the distances of the fiber points from this line is minimal. The crimp diameter is then defined as $D=2\sqrt{S/N}$, where N is the number of points measured along the fiber.

The calculation of S according to this definition involves the solution of third-order equations. Therefore a simplified equation is suggested. It was found that this equation can, for all practical purposes, serve as a satisfactory approximation for the results obtained when S is calculated according to the definition, i.e., from the fiber axis.

In order to measure the crimp diameter, a special apparatus had to be built. A series of limitations of the rotatory crimp apparatus led to the modification of the measuring method and to the development of the method of two perpendicular planes. This method was applied to various types of fiber, such as cotton and fine wools.

Introduction

In a previous work [1], a method of defining the crimp of textile fibers was described. The following parameters were suggested as describing the geometry of the crimp of a fiber while it is kept in a static position of stress.

The main parameter used was the *crimp diameter*. This corresponds to the average amplitude of the fiber in space. In order to measure the crimp diameter it is necessary to measure the coordinates of a sufficient number of points on the fibers; from these the crimp diameter can be calculated.

Another parameter, the *wave number*, was taken as a second independent crimp parameter. This parameter can be obtained from the above defined crimp diameter and the stress–strain data of the crimped fiber, in the form of the *crimp ratio*. For the computations of the wave numbers two regular shaped models were used: a circular helix and a sine curve.

Additional information on the crimp can be obtained from the measurements of the stress–strain

[1] This work is a part of a Ph.D. thesis by M. Shiloh, submitted to the Hebrew University, Jerusalem, Israel.

behavior. Measurements of this behavior and the crimp ratio of the fiber may be carried out in various instruments, e.g., the Instron tester. However, in order to measure the crimp diameter a special apparatus is necessary.

The Rotatory Method

The first method used [1] involved a rotatory crimp apparatus and will therefore be referred to as the rotatory method.

In that method the fiber was rotated around an axis passing through its end points. Its image was projected on a screen. During the rotation each point of the fiber describes a circle in the plane perpendicular to the axis of rotation. In the projection, this circular motion will appear as a movement along a straight line, with the "width" of $2r_iM$, where r_i is the distance of the fiber point from the axis of rotation and M the magnification of the projection. Measurements of $2r_i$ were taken at N points at constant distances along the rotation axis. The crimp diameter is then calculated as

$$D = \left(\sum_{i=1}^{N} \frac{(2r_i)^2}{N} \right)^{\frac{1}{2}} \qquad (1)$$

The line connecting the end points of the fiber is used in this method as a fiber axis from which the distances r_i of the fiber points should be measured. Actually, only the distances from the axis of rotation can be measured in this method. Therefore, much care has to be taken in bringing the end points of the fiber exactly on the rotation axis, and inaccuracies in this adjustment will influence seriously the resulting calculated crimp diameter.

In order to facilitate and speed up this adjustment of the end points on the rotation axis, the apparatus was further developed and improved. This was described in a subsequent paper [2]. However, the measurements were still too time consuming, especially considering the large number of measurements which are necessary in order to represent various samples of crimped fibers. The jaws had to be recentered each time they were opened for the application of additional loads to the fibers. A small hook had to be tied to one end of each fiber, on which additional loads were to be applied. All this required much time. Furthermore, the procedure of centering the jaws was not always successfully completed in practice: the movements of the motor and gears caused slight vibrations so that some surface of revolution of the axis was always present and could not be completely avoided. These inaccuracies had a negligible effect on the measurements of the wool and jute fibers used in the earlier work, but had some influence on the measurements of cotton and fine wool samples which were carried out later on.

Also, the illumination of the fiber in this instrument was too poor for cotton fibers, as the light was passing through the two mirrors; a higher intensity of illumination would have heated the fibers too much.

Another disadvantage of the rotatory method is the error which is caused by the random choice of fiber end points. When the fiber is inserted into the jaws, the point of contact is at a random phase of the wave; this point is used to define the axis of rotation around which the $2r_i$ values are determined. If, on the other hand, some other (random) point is chosen, the axis will generally be a different one; other width values might then be measured and a different value of D be obtained.

When all these limitations were considered, it seemed that an improvement of the method was necessary if fibers with smaller crimp diameters, such as cotton, were to be investigated. Therefore the following method of two perpendicular planes was developed.

Theoretical Considerations

The definition of the crimp diameter given earlier depends on the fiber end points which are chosen for the definition of the axis of measurement.

The following definition of the crimp diameter avoids this ambiguity. As the fiber axis from which the distances r_i are to be calculated, we define that straight line for which the sum of squares of the distances of the fiber points from it will be *minimal*. Let S be this minimal sum of squares; D will be now defined as

$$D = 2(S/N)^{\frac{1}{2}} \qquad (2)$$

D describes approximately the volume occupied by the fiber in space, i.e., the diameter of an "average cylinder" which is defined by the fiber.

The line passing through the end points of the fiber (which is kept in a static state) is chosen as the axis of measurement. The coordinates x_i and z_i at N points of the fiber projection are measured. z is the coordinate along the axis of measurement and x is perpendicular to z.

A corresponding set of measurements (y, z) is taken in a second plane perpendicular to the plane (x, z). All distances $z_i - z_{i+1}$ are chosen to be equal.

These measurements are performed by imposing the fiber pictures on millimetric paper in such a way that z is coincident with one coordinate of the millimetric paper, as shown in Figure 1.

The Calculation of S

According to its definition, the calculation of S involves a least-squares analysis, which is complicated because three dimensions are involved.

For the calculation of S let us choose a set of points $P_i(x_i, y_i, z_i)$, let L be any straight line, and let us define the sum

$$S_L = \sum_{i=1}^{N} r_{iL}^2 \tag{3}$$

where r_{iL} is the distance of the point P_i from the line L. The value of S_L depends upon the position chosen for the line L. S is defined as the minimum of S_L. Any straight line is given by

$$(x - a)/m = (y - b)/n = (z - c)/p \tag{4}$$

All measurements are taken with loaded fibers, the unloaded state being unsuitable for practical measurements. The r_i are always small compared to the distance between the fiber end points. Therefore, it can be assumed that the minimal axis (fiber axis) will not be parallel to the (x, y) plane and therefore we can choose in Equation 4 $c = 0$ and look upon the group of lines which fulfill

$$(x - a)/m = (y - b)/n = z/p \quad \text{where } p \neq 0. \tag{5}$$

We can always choose

$$m^2 + n^2 + p^2 = 1 \tag{6}$$

when m, n, p are the cosines of direction of the line L. For the sum S_L we shall then get

$$S_L = \sum_{i=1}^{N} \{[p(y_i - b) - nz_i]^2 + [mz_i - p(x_i - a)]^2 + [n(z_i - a) - m(y_i - b)]^2\} \tag{7}$$

S_L depends on the parameters a, b, m, n, p which are also linked to the condition in Equation 6. It is now necessary to find those values of these parameters which will give a minimum of S_L.

In order to find the stationary points of the function $S(a, b, m, n, p)$ we shall use the method of

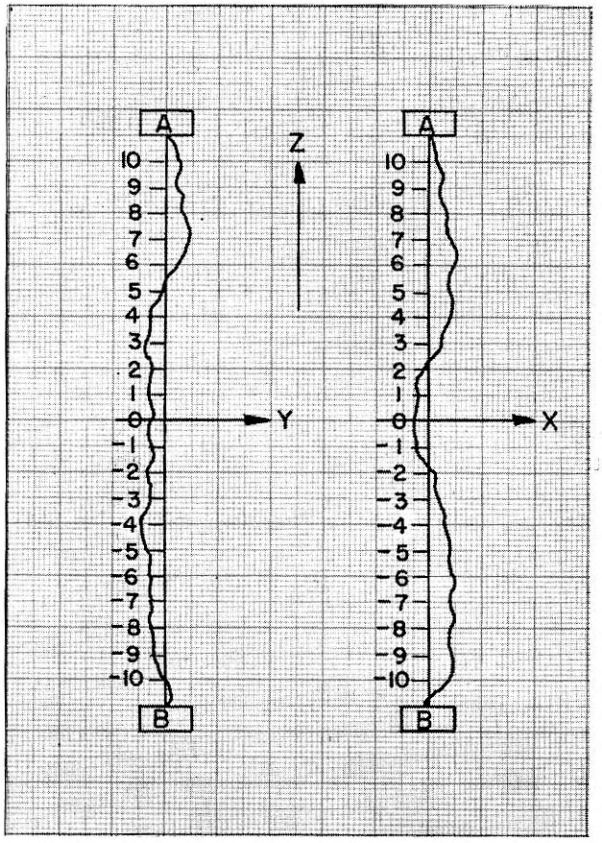

Fig. 1. The method of reading the coordinates of the points of the fiber.

Lagrange's multipliers. We define the function F

$$F(a, b, m, n, p) = S(a, b, m, n, p) - \lambda(m^2 + n^2 + p^2 - 1) \tag{8}$$

together with the condition in Equation 6. The minimum of F will also be the minimum of S_L. From $\partial F/\partial a = \partial F/\partial b = 0$ we get

$$a = \frac{p\sum x_i - m\sum z_i}{Np} \qquad b = \frac{p\sum y_i - n\sum z_i}{Np} \tag{9}$$

In order to get simpler equations for m, n, p we now choose new coordinates (also called x_i, y_i, z_i), so that the origin will be the center of gravity of the system of the N points, or

$$\sum x_i = \sum y_i = \sum z_i = 0 \tag{10}$$

so that

$$a = b = 0 \tag{11}$$

With this assumption we get as a necessary condition of minimum

$$\partial F/\partial m = \partial F/\partial n = \partial F/\partial p = 0 \tag{12}$$

and from Equation 12 the following equations for m, n, p are obtained

$$m[\sum(z_i^2 + y_i^2) - \lambda] + n(-\sum x_i y_i) + p(-\sum x_i z_i) = 0$$

$$m(-\sum x_i y_i) + n[\sum(z_i^2 + x_i^2) - \lambda] + p(-\sum y_i z_i) = 0$$

$$m(-\sum x_i z_i) + n(-\sum z_i y_i) + p[\sum(y_i^2 + x_i^2) - \lambda] = 0 \quad (13)$$

These form a set of homogeneous equations which have a nontrivial solution (other than $m = n = p = 0$) only in the case when the determinant of the coefficients is equal to zero, or

$$\Delta = \begin{vmatrix} \sum(z_i^2 + y_i^2) - \lambda & -\sum x_i y_i & -\sum x_i z_i \\ -\sum x_i y_i & \sum(z_i^2 + x_i^2) - \lambda & -\sum y_i z_i \\ -\sum x_i z_i & -\sum z_i y_i & \sum(y_i^2 + x_i^2) - \lambda \end{vmatrix} = 0 \quad (14)$$

The equation $\Delta = 0$ is the well-known secular equation. Because of the symmetry of the determinant, it is known that there are always three real solutions, λ_1, λ_2, λ_3. For each λ_k ($k = 1, 2, 3$) we get from Equations 13 together with the condition in Equation 6, a solution for (m_k, n_k, p_k). Therefore, there will always be three solution lines which will give the sums S_1, S_2, S_3. The necessary S will be the minimum:

$$S = \min(S_1, S_2, S_3) \quad (15)$$

It can also easily be seen that when Equations 13 are multiplied by m, n, and p respectively and summed together, we get $S = \lambda$; the solutions λ_k therefore give S directly, so that it is not necessary to calculate m, n, and p.

The Approximated Calculation of S

The actual calculation of S which involves the solution of a third-order equation in each case, is too laborious for practical purposes, considering the large number of calculations which are necessary in order to represent various fiber samples. It was therefore decided to calculate S through an explicit equation which may be referred to as an approximated equation.

Let us assume (and this assumption can be well justified by various experiments on crimped fibers) that the minimal axis (fiber axis) and the axis of measurement, z, are intersected an an angle φ which is small, so that

$$r_i = d_i \cos\varphi \simeq d_i$$

where r_i are the distances of the points $P_i(x_i, y_i, z_i)$ to the fiber axis and d_i are the projections of r_i on the (x, y) plane, as shown in Figure 2. Instead of the sum

$$S_L = \sum_{i=1}^{N} r_{iL}^2 \quad (3)$$

we shall now calculate

$$S_L \simeq S_L' = \sum_{i=1}^{N} d_{iL}^2 \quad (16)$$

Assuming that $p \neq 0$ we can now describe the line L by

$$(x - a)/m = (y - b)/n = z/1 \quad (17)$$

or

$$x = mz + a$$
$$y = nz + b \quad (18)$$

It is easily seen that

$$d_{iL}^2 = (mz_i + a - x_i)^2 + (nz_i + b - y_i)^2 \quad (19)$$

and the sum

$$S_L = S(m, n, a, b) = \sum d_{iL}^2 = \sum[(mz_i + a - x_i)^2 + (nz_i + b - y_i)^2] \quad (20)$$

m, n, a, b can be obtained from the necessary conditions of minimum for S_L

$$\begin{aligned} \tfrac{1}{2}\partial S_L/\partial m &= m\sum z_i^2 + a\sum z_i - \sum x_i z_i = 0 \\ \tfrac{1}{2}\partial S_L/\partial n &= n\sum z_1^2 + b\sum z_i - \sum y_i z_i = 0 \\ \tfrac{1}{2}\partial S_L/\partial a &= Na + m\sum z_i - \sum x_i = 0 \\ \tfrac{1}{2}\partial S_L/\partial b &= Nb + n\sum z_i - \sum y_i = 0 \end{aligned} \quad (21)$$

From Equation 21 we get

$$m = \frac{N\sum x_i z_i - \sum x_i \sum z_i}{N\sum z_i^2 - (\sum z_i)^2} = \frac{\sum(x_i - \bar{x})(z_i - \bar{z})}{\sum(z_i - \bar{z})^2}$$

$$n = \frac{N\sum y_i z_i - \sum y_i \sum z_i}{N\sum z_i^2 - (\sum z_i)^2} = \frac{\sum(y_i - \bar{y})(z_i - \bar{z})}{\sum(z_i - \bar{z})^2}$$

$$a = \bar{x} - m\bar{z} \qquad b = \bar{y} - n\bar{z} \quad (22)$$

It is therefore seen that the projection of the fiber axis (minimal axis) on the (x, z) plane, $x = mz + a$, is the regression line of the projections of the N given points on the (x, z) plane, and that

$y = nz + b$ is the corresponding regression line in the (y, z) plane.

From Equation 22 we get the following expression for S

$$S = \sum(x_i - \bar{x})^2 + \sum(y_i - \bar{y})^2 - \left[\frac{[\sum(x_i - \bar{x})(z_i - \bar{z})]^2 + [\sum(y_i - \bar{y})(z_i - \bar{z})]^2}{\sum(z_i - \bar{z})^2}\right] \quad (23)$$

or, using the notation used in statistics, we get

$$S = \text{var}(x) + \text{var}(y) - \left[\frac{[\text{cov}^2(x,z) + \text{cov}^2(y, z)]}{\text{var}(z)}\right] \quad (24)$$

In the actual measurements it is always possible to choose $\sum z_i = 0$ (as shown in Figure 1) so that the equation can be further simplified

$$S = \sum(x_i - \bar{x})^2 + \sum(y_i - \bar{y})^2 - \frac{[(\sum x_i z_i)^2 + (\sum y_i z_i)^2]}{\sum z_i^2} \quad (25)$$

and usually $\sum z_i^2$ may be constant for a certain series of measurements.

In these equations S can always be divided into two individual sums: $S = S_x + S_y$. These sums can be calculated from the (x, z) and (y, z) planes separately.

S is then introduced to Equation 2 in order to get the crimp diameter

$$D = 2(S/N)^{\frac{1}{2}} \quad (2)$$

A Numerical Example

The difference between the sum S and the approximated sum was calculated from the measured coordinates of various fibers and under various stresses. One of the most outstanding differences was obtained in the following case.

The coordinates x and y (in mm.) were measured from two perpendicular projections of a cotton fiber loaded with 2.5 mg. and with an enlargement of 31× (Table I). The value of S is found to be $1103.4/31^2$ (mm.^{2}) and $D = 0.479$ mm. The value of S obtained by the approximated equation is $1110.9/31^2$ (mm.^{2}) and $D = 0.481$ mm. It is seen that the difference does not exceed 0.7%, so that it is negligible when compared to the desired accuracy of the measurements.

The use of the approximated equation for S is therefore justified.

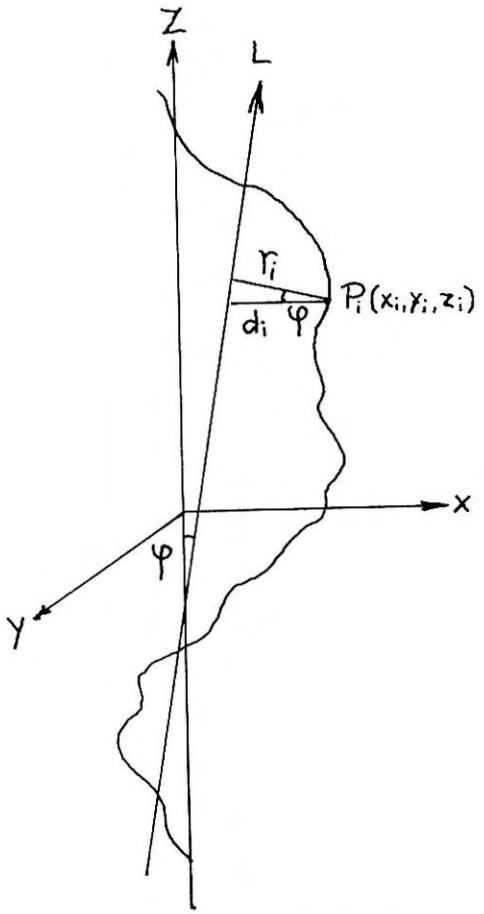

Fig. 2. The fiber axis and the axis of measurement.

TABLE I.

z	x	y
10	16.8	−14.8
9	4.2	8.0
8	3.0	−11.1
7	4.0	−9.8
6	10.0	−1.3
5	10.3	3.0
4	3.0	−1.2
3	−0.5	0.4
2	5.5	5.1
1	10.1	5.0
−1	−1.5	−0.6
−2	−2.7	−1.1
−3	2.0	−2.5
−4	−2.3	6.0
−5	9.0	−9.0
−6	3.4	−8.3
−7	1.5	−12.3
−8	0.6	−5.8
−9	0	−4.0
−10	0	5.4

Experimental

The New Crimp Meter

According to the new definition of the fiber axis, it is not necessary to rotate the fiber; instead of rotation we get two perpendicular projections of two planes of the fiber.

The measurements of the extensions and the crimp ratio are carried out in the same manner as in the rotatory apparatus, but the crimp diameter is obtained from the measurements on each plane separately.

Let the z axis be the straight line which passes through the end points on each plane. These end points are the only marked points along the fiber and therefore allow the simultaneous choice of the z axis on both planes. Each point $P_i(x_i, y_i, z_i)$ of the fiber will describe the points (x_i, z_i) and (y_i, z_i) on both projection planes. From these projected points the minimal sums of squares, S_x and S_y, may be calculated, and the crimp diameter may be obtained

$$D = 2(S/N)^{\frac{1}{2}} = 2(S_x/N + S_y/N)^{\frac{1}{2}}$$

by using Equations 23 or 24.

The projections can be obtained on screens at various distances from the fiber and at various enlargements. Figure 3 shows the screens which were used in the present work on cotton fibers, at a distance of 3 m. from the fiber with an enlargement of 20×. In order to save space, one projection was directed through a mirror at 45° to the beam of light, so that both projections were obtained on one table. The mirror is also shown in Figure 4.

In Figure 4 the apparatus which has been built for this method of measurement is shown. In this apparatus the upper fiber end is fastened to a small jaw which is attached to a steel rod. This rod is hooked to a torsion balance. The lower end of the fiber is inserted into a jaw which is connected to a moving table. The movement of the table which carries the jaw is controlled by the lower handle, which is shown in the figure. Two lenses are rigidly connected to this table on two perpendicular arms. At the back part of the instrument, opposite to the arms, two sodium discharge lamps illuminate the fiber so as to cause the formation of the images of two of its planes through the lenses on the screens. To the moving table (which can be

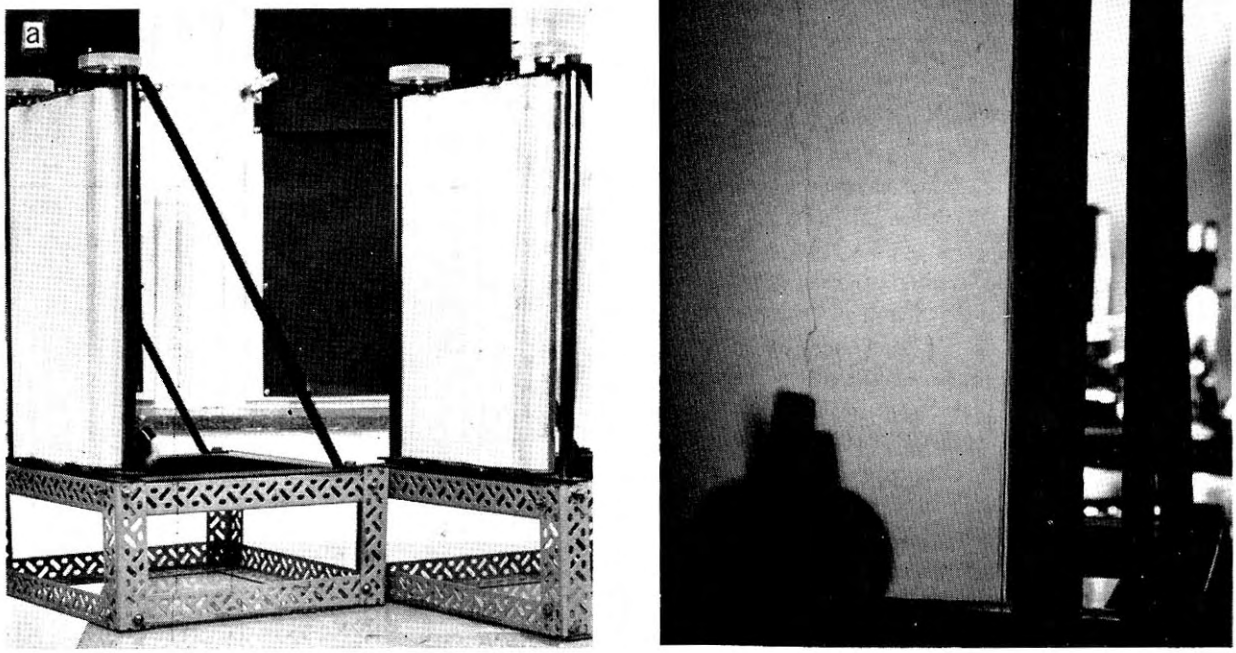

Fig. 3. The projection screens: a, a general view; b, the projection of the fiber on one plane.

lifted or lowered with the aid of the screw under the apparatus, as shown in Figure 4) a millimetric gauge is connected; on this gauge, the movement of the table, which corresponds to the stretching of the fiber, can be read and controlled. It is possible to counterweight the torsion balance ("Hartman and Braun," full scale reading of 2500 mg., maximum load 25 g.) against the weight of the rod and the jaw, on which the upper end of the fiber is attached. When the fiber is stretched, the weight which is necessary to lift the rod can be measured on the balance; this weight is equivalent to the load which caused the stretching of the fiber. In this way the static stress–strain characteristics of the fiber can be obtained, starting at 5 mg. and ending at breakage.

This method of mounting and measuring proved far more satisfactory than the previous method where weights had to be attached to the fibers and the extensions measured by a comparator.

The Mounting of the Fibers

A special method of gluing the fibers has been developed, in order to reduce handling of the fibers and to simplify the measurements on the crimp meter. Celluloid sheets, of a thickness of 0.5 mm., have been cut in the form shown in Figure 5.

The fibers are glued at the points A and B by an organic solution which dissolves the celluloid in the neighborhood of the points of contact. In this way slippage of fibers in the jaws can be avoided.

This frame is later cut (at the dotted lines C, D) into individual frames. For the measurement, a single frame is hung on the upper jaw of the crimp meter, the lower jaw of the apparatus is then closed on the lower end, B, of the frame. Now the vertical sides of the frame are cut away. From this position the fiber can be subjected to crimp measurements at various controlled extensions.

This method of fastening the fibers is also suitable for the tests of these and other fibers on the Instron tester, in order to get their stress–strain

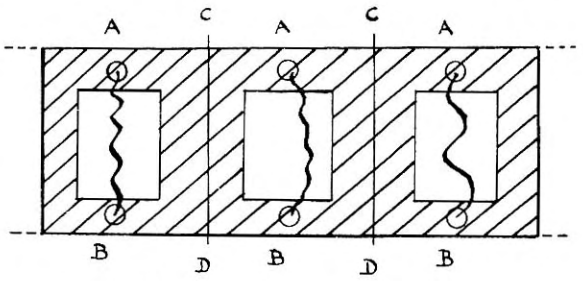

Fig. 5. The mounting of the fibers; fibers are glued to the frame.

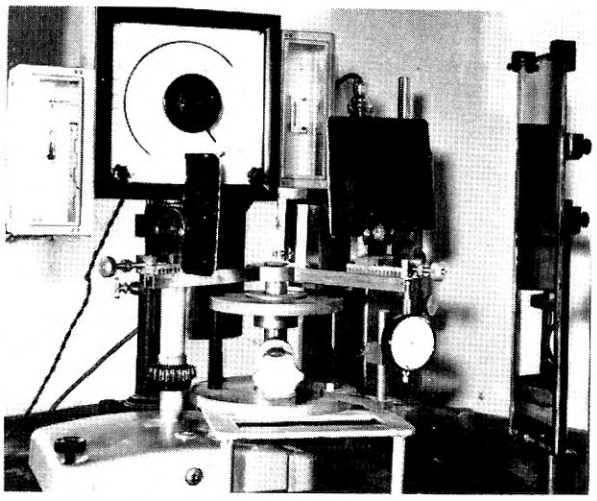

Fig. 4. The crimp meter.

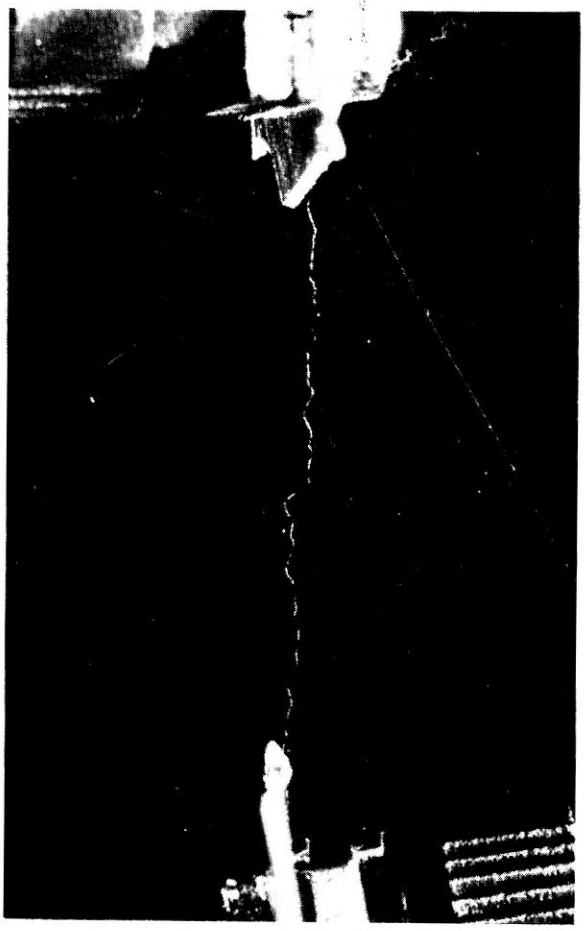

Fig. 6. A crimped cotton fiber (5×).

continuous curves and their uncrimping energies.[2] The method of fastening the fibers to the frame can be observed in Figure 4. Figure 6 shows a cotton fiber which was glued to such a frame and photographed on the Instron tester.

The Procedure of the Measurement

After the fiber has been inserted into the crimp meter, the lenses are set up for the appropriate magnifications of the projections on the screens. The points A and B, where the fiber is glued to the frame, are easily recognized on the screens because of the transparency of the celluloid; these points are therefore chosen as the ones defining the axis of measurement, z, as they ensure the same axis on both (x, z) and (y, z) planes, and there are no other marked points along the fiber.

Transparent drawing paper is used in order to copy the projection images on both screens. These drawings are later used in order to read the the coordinates; each drawing is put upon millimetric paper in the way shown in Figure 1. The end points A and B are directed so as to be on top of a marked line of the millimetric paper which will serve as the z axis. A line which is perpendicular to this line at the center of the portion AB is defined as the x or y axis. Then the deviations x_i or y_i from the z axis are read at N constant intervals of z; these values are later introduced into Equations 23, 24, or the simplified Equation 25. From this S and therefore D are obtained.

[2] The comparison of the results obtained for the uncrimping energies with the data of D for the same fibers will be given in a subsequent work.

Conclusions

The method of two perpendicular planes of a fiber has proved to be a simplified and more accurate method of measuring the crimp diameter of various fibers.

The use of the approximated equation for the minimal sum of squares of the distances of the fiber points to the fiber axis is justified for all practical purposes.

The same equations can also be used to measure the sum of squares of the deviations of any set of points from their regression line.

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